

Some encouraging and some cautionary remarks on Doubly Special Relativity in Quantum Gravity¹

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ABSTRACT

The idea of a role for DSR (doubly-special relativity) in quantum gravity finds some encouragement in a few scenarios, but in order to explore some key conceptual issues it is necessary to find a well-understood toy-quantum-gravity model that is fully compatible with the DSR principles. Perhaps the most significant source of encouragement comes from the recent proposal of a path for the emergence of DSR in Loop Quantum Gravity, which however relies on a few assumptions on the results of some computations that we are still unable to perform. Indications in favor of the possibility of using some elements of κ -Poincaré Hopf algebras (and of the related κ -Minkowski noncommutative spacetime) for the construction of a DSR theory have been discussed extensively, but a few stubborn open issues must still be resolved, especially in the two-particle sector. It has been recently observed that certain structures encountered in a formulation of 2+1-dimensional classical-gravity models would fit naturally in a DSR framework, but some key elements of these 2+1-dimensional models, including the description of observers, might be incompatible with the DSR principles.

1 Introduction

In Doubly-Special Relativity[1] (DSR) one looks for a transition in the Relativity postulates. A transition which should be largely analogous to the Galilei \rightarrow Einstein transition. Just like it turned out to be necessary, in order to describe high-velocity particles, to set aside Galilei Relativity (with its lack of any characteristic invariant scale) and replace it with Special Relativity (characterized by the invariant velocity scale c), it is at least plausible that, in order to describe ultra-high-energy particles, we might have to set aside Special Relativity and replace it with a new relativity theory, a DSR, with two characteristic invariant scales, a new small-length/large-momentum scale in addition to the familiar velocity scale.

A theory will be compatible with the DSR principles if there is complete equivalence of inertial observers (Relativity Principle) and the laws of transformation between inertial observers are characterized by two scales, a high-velocity scale and a high-energy/short-length scale. There are of course many observer-independent scales in physics (the Planck constant \hbar , the electron mass (rest energy) m_e , ...) but from the perspective of the Relativity

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Principle in our present theories there is one observer-independent scale which plays a very special role: the speed-of-light scale c appears in the rules of transformation² between inertial observers. In a DSR theory one would have also a second observer-independent scale with this property, a short length scale λ (large momentum scale $1/\lambda$).

Since in DSR one is proposing to modify the high-energy sector, it is safe to assume that the present operative characterization of the velocity scale c would be preserved: c is and should remain the speed of massless low-energy particles. Only experimental data could guide us toward the operative description of the second invariant scale λ , although its size is naturally guessed to be somewhere in the neighborhood of the Planck length. And, of course, it is very difficult to test the DSR idea in very general terms. There are a few predictions that can be obtained using only the general structure of the DSR Relativity principles. For example, whereas in schemes for Planck-scale-broken Lorentz symmetry one naturally finds[2, 3, 4] the characteristic prediction that photons become unstable at sufficiently high energies, in a DSR framework one may find that the second observer-independent scale λ affects the lifetimes of some particles, but necessarily one must³ have in DSR that a particle which is stable at low energies is also stable at any however high energy. However, with the exception of a few universal predictions (such as this one pertaining to particle stability/instability), different realizations of the DSR principles (in particular, different postulates attributing operative meaning to the second observer-independent scale λ) will in general lead to different physical processes. So one must in principle consider all possible formalisms and physical pictures that could provide a realization of the DSR idea.

Primarily inspired by a certain analysis of some preliminary astrophysics data[5, 6, 7], the most studied[1, 8, 9, 10, 11] possibility assumes that the second invariant λ should acquire operative meaning by appearing in the dispersion relation (the relation between energy and momentum of a particle of mass m), which could take the form $E = f(p, m; c, \lambda)$. The to-be-determined function f must of course be such that for small momenta $f \simeq \sqrt{c^2 p^2 + c^4 m^2}$.

In this specific type of DSR framework, which involves a λ -modified dispersion relation, the “discovery of DSR” would probably emerge following a path which is analogous to the one of the discovery of Special Relativity. The Michelson-Morley experiments can be viewed as experiments providing evidence in favor of a c -modification of the Galilei dispersion relation $E = p^2/2m$. At first the c -modification was interpreted as a violation of the Relativity Principle (*i.e.* a manifestation of the existence of a preferred class of inertial observers, an ether), but then it was established that the c -modification is observer independent: the Lorentz boost transformations are a modification of the Galilei boost transformations such that the dispersion relation $E = \sqrt{c^2 p^2 + c^4 m^2}$ holds in every inertial frame. If evidence in favor of a dispersion relation of type $E = f(p, m; c, \lambda)$ does emerge it would be natural to guess that λ characterizes a preferred class of inertial observers, but one should also con-

²The Relativity Principle prescribes that the laws of physics are the same for all inertial observers, but of course the same process is in general characterized by different measured quantities in two different frames. The quantitative differences between the measurement results of the two observers are connected through the laws of transformations between inertial observers.

³If in a given theory, say, a photon is stable at low energies but becomes unstable when its energy is higher than a certain value E_* then clearly the theory is being formulated for a specific class of observers. Two observers typically disagree on the energy of a particle but should not disagree on whether or not a particle decays.

sider the possibility[1] that instead the boost transformations might be λ -modified without affecting in any way the Relativity Principle. Of course, if the boost transformations are modified in order to render invariant a dispersion relation of the type $E = f(p, m; c, \lambda)$ then one has repercussions also elsewhere in the relativistic theory, and in particular the laws for the conservation of energy and momentum must be also modified[1, 10] (in order to preserve their status as covariant laws under boosts).

From a quantum-gravity perspective these DSR scenarios with modified dispersion relation can be rather attractive, especially because of the possibility to introduce a maximum momentum $1/\lambda$ (for example[1, 8, 10] through a dispersion relation such that $p \rightarrow 1/\lambda$ as $E \rightarrow \infty$). And several attempts to introduce/find DSR in various descriptions of quantum gravity and/or quantum spacetime have been made over these past 3 years. Here I comment on some of these studies and describe some possible general lessons that can be drawn from them.

2 The illustrative example of κ -Minkowski noncommutative spacetime

Already the first studies of the DSR idea[1, 8] explored the possibility that in the κ -Minkowski[12, 13] noncommutative spacetime, characterized by coordinate commutation relations $[x_j, t] = ix_j/\kappa$, $[x_j, x_k] = 0$, one might be able to construct theories compatible with the DSR principles. Some results obtained in the analysis of κ -Minkowski do encourage the possibility that the laws of transformation between inertial observers should be of a DSR-compatible type, with $\lambda \equiv 1/\kappa$. It had been known for some time that κ -Minkowski is dual (in an appropriate algebra sense) to the much-studied κ -Poincaré[13, 14] Hopf algebras. Even though there are still some grey areas[15] in the physical characterization of a Hopf-algebra description of spacetime symmetries, this duality and the structure of the relevant Hopf algebra can provide encouragement for a modification of boosts that is admissible in DSR.

While the preceding κ -Poincaré Hopf-algebra literature had warned⁴ against[16] attempts to integrate the boost generators to obtain a candidate for finite boost transformations, the attempts[1, 8] to obtain a DSR physical theory on the basis of κ -Poincaré mathematics could not avoid a description of finite boosts, and in the investigation of these issues it emerged that some descriptions of the κ -Poincaré Hopf algebra do allow (by exponentiation of the κ -Poincaré generators) a perfectly consistent implementation of finite boosts on one-particle systems. And these recent investigations also showed that it is rather natural to introduce a maximum value for momentum (and/or energy), *i.e.* to obtain a description of boosts that saturates to the maximum value $p_{max} = \kappa = 1/\lambda$. The picture of energy-momentum space admits description in terms of a deSitter-type geometry[18], at least in the sense that one can find a straightforward map between the energy-momentum variables and the coordinates of a deSitter-type spacetime.

⁴Ref. [16] observed that by exponentiating κ -Poincaré generators one would in general not obtain a symmetry group (only a “quasigroup” in the sense of Batalin[17]).

Because of the mentioned duality, these features of κ -Poincaré should be applicable to theories in κ -Minkowski spacetime, and this is of encouragement for a DSR formulation of physics in κ -Minkowski. But the duality with κ -Poincaré is also the source of some concern for a DSR description of multi-particle systems in κ -Minkowski. In fact, while in the one-particle sector everything can be formulated in a DSR-compatible way (*i.e.* one can find κ -Poincaré mathematical structures which can be used to construct a DSR physical theory), already for a simple system of two particles there is no known way to use κ -Poincaré mathematics to obtain a DSR-compatible formulation. In particular, the law of energy-momentum conservation which is advocated by κ -Poincaré experts[19] is incompatible with the DSR principles, since it combines with the κ -Poincaré dispersion relation in such a way to lead to the emergence of a preferred class of inertial observers[20, 4]. Perhaps the solution of this problem will simply require us to uncover some new structures within the κ -Poincaré Hopf algebra mathematics, but at present this can only be conjectured.

3 Proceeding with caution

κ -Minkowski has been the first example of quantum spacetime considered from a DSR perspective. Although, as just stressed, it is still unclear whether κ -Minkowski really is fully compatible with the DSR principles, it is a good test case to explain how DSR-type structures might emerge in a quantum spacetime. One way to see this is through the analysis of plane waves in κ -Minkowski, $e^{ikx}e^{-ik_0t}$, where it is clear that the composition of two waves, $e^{iqx}e^{-iq_0t} = e^{ikx}e^{-ik_0t}e^{ipx}e^{-ip_0t}$, cannot involve a linear combination of momenta ($q = k + p$ is incompatible with the relations $[x_j, t] = ix_j/\kappa$, $[x_j, x_k] = 0$). If κ -Minkowski is eventually understood as a DSR-compatible spacetime it is likely that this nonlinear law of composition of wave exponentials will play a key role in the analysis.

A few other examples of quantum gravity or quantum-spacetime pictures which invite one to consider the DSR possibility have been found. But at present the key challenge for this research programme is to show that at least in one specific context DSR is actually present, a specific example of a framework that is fully compatible with the DSR principles. As presently understood κ -Minkowski, because of the key issues that remain unsettled, still cannot be viewed as such a fully-worked-out DSR example. And among the other “DSR candidates” the situation is similar: some features that provide support for the idea of a DSR description have been uncovered, but the full compatibility with the DSR principles has not been established. We are however learning more about what is needed and what is not needed for (or is incompatible with) a DSR framework, and in setting up future research on this subject these lessons might be precious.

3.1 DSR in classical spacetime not likely

If indeed one is looking for the specific type of DSR framework which involves nonlinearities in the energy-momentum sector it seems unlikely that one should be able to construct the theory in a classical spacetime (*i.e.* a spacetime with sharp localization of events). In a flat classical spacetime plane waves will combine in the familiar straightforward way (unlike the case of κ -Minkowski noncommutativity, which, as mentioned, inevitably leads to nonlinearities of the

composition of waves). In such contexts nonlinearities in the transformation laws of energy and momentum could be introduced only in a rather fictitious way (*e.g.* by an otherwise unjustified redefinition of the energy and momentum variables starting from the standard special-relativistic case), and, chances are, at some point the formalism will remove the nonlinearities at the level of truly observable predictions.

Even if one attempts to introduce the two scales directly in spacetime structure (rather than starting from an energy-momentum space intuition) some difficulties should emerge. Some authors have placed much emphasis of the fact that Fock, long ago[21], stumbled upon the observation that in a flat spacetime one obtains transformation laws that involve two scales upon renouncing to the objectivity of parallelism of worldlines. Fock was actually thinking in reverse: in an analysis of the conceptual structure of Special Relativity he explored the implication of the removal of one of the hypothesis of Special Relativity, the one that establishes that two worldlines are parallel for all observers if they are parallel for one observer, and found that this would allow a two-scale family of transformation laws. But, while conceptually insightful, this result did not gain any interest from a physics perspective (not even in the eyes of Fock himself[21]) since, as one should expect, the second scale ended up being necessarily a large length scale (removed in the large-distance limit rather than the short-distance limit), a possibility which can be safely excluded on the basis of our abundant low-energy data, which are all fully compatible with ordinary (one-invariant-scale) Special Relativity. It seems that essentially Fock might have rediscovered deSitter spacetime: his work started from the assumption of a flat spacetime but by removing the objective parallelism of worldlines, and assuming that spacetime could be described in terms of a classical geometry, he effectively introduced constant curvature in spacetime.

3.2 Hopf-algebra description of Poincaré symmetries not sufficient for DSR

As stressed in the previous section, when I considered the possibility of obtaining a DSR theory in κ -Minkowski spacetime using κ -Poincaré mathematics, a two-scale Hopf-algebra generalization of the Poincaré Lie algebra does not automatically provide the mathematical ingredients to formulate a DSR theory. Perhaps as a result of the focus on mathematics that permeates all of quantum-gravity research this point is often missed. An algebra involves a certain collection of mathematical structures which admit in principle a large number of possible uses in the construction of physical theories. The example of the κ -Poincaré Hopf algebra should help clarify this point: there are plenty of structures in the κ -Poincaré/ κ -Minkowski literature (*e.g.* the generators, the coproducts, the antipodes...) but in physics one must find a way to use these structures in the description of various physical features of the relativistic theory.

In the simplest cases the mathematics just confronts us with some alternatives, and the physicists select the structures on the basis of compatibility with certain physics principles. For example in the description of fields in κ -Minkowski one must of course introduce a differential calculus, for which various alternatives have been proposed in the κ -Minkowski literature. In particular there has been interest in a four-dimensional[22] and in a five-dimensional[23] differential calculus, and both of these differential calculi deserve equal

consideration at the mathematics level, but if one insists on compatibility with the DSR principles only the five-dimensional differential calculus turns out [24] to be acceptable.

And one cannot exclude that among the many alternatives provided by the mathematics side there might not be a single one that accomplishes the tasks required by the physicist. For example in the search of a DSR-compatible law of energy-momentum conservation physicists should choose among a large variety of different structures on the mathematics side which are plausible candidates as building blocks for the law of energy-momentum conservation. And, as mentioned, it is still unclear whether any of the mathematical structures which emerged in the κ -Poincaré literature would allow to introduce a law of conservation of energy-momentum that is compatible with the DSR principles (at least it is at present not known that any of these structures would lead to a DSR-compatible conservation law).

3.3 deSitter geometry for energy-momentum space not sufficient for DSR

Just like a superficial look at κ -Minkowski might lead to the incorrect expectation that the availability of a two-scale Poincaré-like algebra would be automatically sufficient for a DSR formulation, in turn the observation that κ -Poincaré, in an appropriate sense, predicts an energy-momentum space which has deSitter-type geometry might lead to the incorrect expectation that whenever it is possible to find a natural-looking map from the energy-momentum variables to some coordinates over a deSitter geometry one should be able to find a DSR formulation of the theory. Of course, this expectation is also incorrect. It is true that a natural way to introduce a second relativistic scale, in the sense of DSR, can be the one of an energy-momentum space with observer-independent constant curvature, but the observer-independence of the curvature is not assured by the existence of a map connecting the energy-momentum variables (as measured by a given observer) to some coordinates on a deSitter geometry.

In order to make this remark more concrete let me propose a simple analogy. The propagation of light in a water-pool is (to very good approximation) described by a dispersion relation $E = \sqrt{c_{water}^2 p^2 + c_{water}^4 m^2}$ (of course, $m = 0$ for photons) which of course allows a map from the energy-momentum variables to some coordinates on a Minkowski geometry. But we know that the scale c_{water} is not observer independent.

From the DSR perspective the problem is even more serious: assuming the space of one-particle energy-momentum is truly deSitter-like (with observer-independent curvature) it remains to be seen how the space of multiparticle momenta should be described. If indeed the DSR proposal involves nonlinearities in energy-momentum space the step from the one-particle sector to the multi-particle sector will inevitably involve some delicate issues.

The fact that the presence of a deSitter-type geometry for energy-momentum space cannot be used to fully specify the symmetry properties was already rather clear to Snyder in the 1940s. Snyder was looking[25] for a nonclassical description of spacetime which would regulate the UV divergences of quantum field theory while being fully compatible with ordinary⁵

⁵This renowned paper by Snyder[25] is one of the most cited in the noncommutative-geometry literature, but apparently it is often cited without reading, and this is leading to frequent misrepresentations of the objectives and the results of the paper. For example, it is sometimes said that Snyder proposed a modification

(undeformed) Lorentz symmetry, and Snyder used a strategy which automatically implied that energy-momentum space would have a deSitter-type geometry.

4 Special caution in 2+1-dimensional spacetime

Two recent papers[26, 27] have considered the possibility that the description of gravity in 2+1-dimensional spacetime might provide a good toy model in which to construct a DSR theory. Before discussing these proposals I find it necessary to stress that from a DSR perspective the 2+1 context might not provide the correct intuition for the 3+1 context. In the study of most types of theories the 2+1 context contains the same logical structure as the 3+1 context, but with a welcome reduction in the level of technical complexity. But from a DSR perspective the logical structure of theories in 2+1 dimensions may be significantly different from the one of their 3+1-dimensional counterparts. This can be seen already at the simple level of dimensional analysis of the scales involved in the theory. Whereas in 3+1 dimensions both the Planck length and the Planck energy are related to the gravitational constant through the Planck constant ($L_p \equiv \sqrt{\hbar G/c^3}$, $E_p \equiv \sqrt{\hbar c^5/G}$), in 2+1 dimensions the (2+1 version of the) Planck energy is obtained only in terms of the speed-of-light scale and the gravitational constant: $E_p^{(2+1)} \equiv c^4/G^{(2+1)}$. (The Planck length however is still introduced through the Planck constant, $L_p^{(2+1)} \equiv \hbar c^3/G^{(2+1)}$.) Since a possible novel operative meaning for the Planck energy and/or length is the key issue under investigation in DSR this observation could be important.

4.1 Cautionary remarks on DSR from 2+1D quantum gravity with q-deSitter symmetry algebra

At least in some formulations of quantum gravity in 2+1 spacetime dimensions a q-deformed deSitter symmetry algebra $SO(3, 1)_q$ emerges[28] for nonvanishing cosmological constant, and the relation between the cosmological constant Λ and the q deformation parameter takes the form $\ln q \sim \sqrt{\Lambda} L_p$ for small Λ . It was observed in Ref. [26] (using a well-established result on the contractions of the q-deSitter algebra[14]) that this relation $\ln q \sim \sqrt{\Lambda} L_p$ implies that the flat-spacetime limit ($\Lambda \rightarrow 0$) is not described by a Poincaré Lie algebra but rather by a κ -Poincaré Hopf algebra. This provided the first ever argument in favor of the possibility of the emergence of DSR in 2+1D quantum gravity. But the analysis only shows that the κ -Poincaré Hopf algebra should have a role in the flat-spacetime limit, without providing a fully physical picture of this role. And in any case, as stressed above, the presence of a κ -Poincaré Hopf algebra somewhere in the formalism of the theory, while providing automatically some ingredients that are suitable for DSR relativity, also introduces some unsolved issues for the consistency with the DSR principles. So the analysis reported in Ref. [26] rigorously shows that the flat-spacetime limit in certain formulations[28] of 2+1D quantum gravity is not described by classical (Lie-algebra) Poincaré symmetries, but, pending the mentioned open

of Lorentz symmetry, and this is rather paradoxical since, on the contrary, Ref. [25] provides a clear statement of objectives, very early in the analysis, in which the preservation of ordinary Lorentz symmetry is stressed as a key point.

issues concerning the possibility of using κ -Poincaré mathematics in the construction of DSR physical theories, it is not really conclusive concerning the emergence of a DSR framework.

4.2 Cautionary remarks on DSR in the classical 2+1D gravity of Matschull *et al*

In the more recent Ref. [27] it was also observed that some aspects of a certain formulation of classical gravity for point particles in 2+1 dimensions, mostly due to Matschull *et al*[29, 30, 31], are compatible with the DSR idea. The key DSR-friendly ingredients are the presence of a maximum value of mass and a description of energy-momentum space which admits mapping onto the coordinates of a deSitter geometry. These are rather striking observations, and Ref. [27] presents an elegant argument which rather compellingly raises the possibility of a DSR formulation of the framework developed by Matschull *et al*. However, as for the proposal discussed in the previous subsection, also in this case several additional results must be obtained in order to verify whether or not such a DSR formulation is possible. One key point is that Matschull *et al* formulate[29, 30] the theory from the very beginning by making explicit reference to a specific frame, the frame of the center of mass of the multiparticle system. There is therefore no natural reason to assume that the features that emerge from the analysis are observer independent. And, as stressed above, features like the presence of a maximum energy and of a deSitter-geometry-compatible structure in energy-momentum space can emerge in equally natural manner in a theory with a preferred frame and in a DSR theory.

Moreover, rather than a deformation of the 6 translation/rotation/boost classical (Lie-algebra) symmetries of 2+1D space, many aspects of the theory, because of an underlying conical geometry, appear to be characterized by only two symmetries: a rotation and a time translation. The framework of Matschull *et al* has a first level of formulation describing (topological) gravity interactions among particles in a classical Minkowski 3D spacetime. But the level of formulation where one is here attempting to see the emergence of a DSR framework is the formulation in which this interacting system is turned into a system of free particles in a gravity-modified geometry (turning dynamics into kinematics by changing geometry). And at that level indeed the framework of Matschull *et al* must be described in terms of conical geometry, characterized only by a rotation and a time translation symmetry transformations.

The description in terms of conical geometry is also closely related to the fact that the “observers at infinity” in the framework of Matschull *et al* do not really decouple from the system under observation. These observers might not be good examples for testing the Relativity Principle.

For the conjecture of emergence of a DSR framework it is also puzzling that, especially when considering particle collisions, Refs. [29, 30, 31] appear to describe frequently as total momentum of a multiparticle system simply the sum of the individual momenta of the particles composing the system. From a DSR perspective, such a linear-additivity law for total momentum is of course incompatible[1] with deformed laws of transformation of energy-momentum. So it appears that this theory of classical-gravity interactions among point particles in 2+1 dimensions, while providing important intuition for other aspects of

quantum-gravity research, is likely to fail to be useful in DSR research, unless we manage to introduce some significantly new elements with respect to the original formulation of Refs. [29, 30, 31].

5 A path for DSR in Loop Quantum Gravity

While I am here placing strong emphasis on some open issues that confront the further development of DSR research, clearly the robust results we already have are very significant and go well beyond what one could have expected[1] as the results of only 3 years of work. And we now even see a truly remarkable opportunity for DSR: as observed in the later part of the analysis reported in Ref. [26] there is a possible path for the emergence of a role for DSR in Loop Quantum Gravity, which is one of the most popular approaches to the quantum-gravity problem.

5.1 The Kodama state, q -deSitter, and DSR

The path for the emergence of a role for DSR in Loop Quantum Gravity proposed in Ref. [26] is closely analogous to the one described here in Subsection 4.1 for one of the formulations of 2+1D quantum gravity. In fact, also in the 3+1D context of Loop Quantum Gravity the literature presents some support for the presence of a q -deformation of the deSitter symmetry algebra when there is nonvanishing cosmological constant. These arguments are based mainly on the properties of the Kodama state[32] and on some approaches to the formulation of boundary observables in Loop Quantum Gravity[33]. As discussed in Ref. [34] (which had argued for a mechanism rather similar to the one then developed in Ref. [26], but without the elements concerning the symmetry-algebra analysis) and Ref. [26], in the 3+1D context one expects a renormalization of energy-momentum which is still not under control in the Loop-Quantum-Gravity literature. For the analysis of Ref. [26] this essentially turns into an inability to fully predict the relation between the q -deformation parameter and the cosmological constant, but for small Λ one should expect $\ln q \sim (\sqrt{\Lambda} L_p)^r$, with r a numerical parameter to be determined through the mentioned renormalization procedure. If the choice $r = 1$ turns out to be correct one would find again (as for the formulation of 2+1D quantum gravity mentioned in Subsection 4.1) that the flat-spacetime limit, $\Lambda \rightarrow 0$, is characterized by a κ -Poincaré Hopf algebra (and in turn, assuming the issues mentioned in Section 2 find a positive solution, this could lead to a DSR formulation).

5.2 A new Planck-scale Cosmology even without DSR

I also want to stress that the hypothesis $\ln q \sim (\sqrt{\Lambda} L_p)^r$ for a q -deformed deSitter algebra in Loop Quantum Gravity is actually interesting even if the choice $r = 1$ turns out not to be correct. For $r > 1$ the flat-spacetime limit would be characterized by a classical Poincaré algebra, but the q -deformation would still be significant, with potentially interesting consequences in phenomenology and cosmology, whenever $\Lambda \neq 0$ (or there is a nonvanishing curvature scalar).

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